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Behavioral modeling in micro simulation models.

A survey*

by

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ABSTRACT

Micro-simulation is an approach to analyze the impact of economic and social policy on the distribution of target variables, not just on the means. It easily includes the true policy instruments and handles highly nonlinear relations. Most models currently used in policy analysis are static and they do not include behavioral response to policy changes, just their first order effects. There is, however, an increasing demand for dynamic models including behavioral responses. This paper surveys current theory and practice in micro-simulation with an emphasis on behavioral modeling, and discusses issues of model building, data availability, estimation, testing and validation.

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1. Micro-simulation, an introduction

Micro-simulation can be viewed as an attempt to model and simulate the whole distribution of policy target variables, not only their mean values. For instance, in a micro-simulation study one might analyze the impact of an income tax change on the whole distribution of income, who loses and who gains. The micro-simulation approach is thus primarily designed for studies of the distributional effects of economic policy, and one of its main advantages is that it permits assumptions of heterogeneous behavior. Every individual, household or firm does not necessarily behave as the average economic agent. This, as a matter of fact, widens the scope of micro-simulation beyond that of conventional econometric modeling. When economic relations are highly nonlinear, when tax laws and rules of transfer programs introduce censoring and truncation and when sub-populations differ in behavior, then models of average behavior become inadequate to evaluate the average impact of policy changes, while a micro-simulation model can be used also for this purpose. A good example is one of the first micro-simulation models actually used for policy evaluation outside academia, namely the model of the Swedish supplementary pension scheme, see Eriksen (1973). For a review see Klevmarken (1973) [1].

Many micro-simulation models identify subgroups or sub-populations each of which are assumed homogeneous in behavior. In these models one only have to simulate the behavior of each subgroup. Population totals and means are then obtained by weighting each group with its relative size. An alternative and usually more general and flexible approach is to simulate each individual, household or firm. In this case the simulation model usually operates on a real sample of individual units. Gradually through the simulation process the sample values of each unit are updated. An advantage of this approach is that the analysis is not limited to certain preselected subgroups of units, but the analyst can choose any mode of analysis of the updated sample. If this sample was a probability

[1] These pensions were based on the earnings of the best fifteen years during a career and to compute future pension obligations and contributions rates a model was needed which could simulate life-time earnings paths and identify the fifteen best years for each individual.

sample from some population, the sampling weights can in principle be applied also to the updated sample for an inference to this population.

In the micro-simulation literature it has become a convention to distinguish between static and dynamic models. In static models the population structure is not updated internally within the model but changes in the composition of the population, for instance in the age distribution, are accounted for by reweighting. Dynamic models, however, include mechanisms, which age the population and allow old population members to leave the population and new members to join. However, models which are dynamic in this sense do not necessarily have a structure which assumes that people's behavior is dynamic, i.e. that past experiences and future expectations influence current decisions.

A limitation of many micro-simulation models is that they only include the rules, which determine the outcome of economic policy, for instance the tax rules and tax schedules, but no behavioral relations. These models can thus only be used to simulate the first order effects of policy changes. The adjustment effects, which follow because people change their behavior as a result of the policy changes, are ignored. Unfortunately we know little about their relative importance. How one could best extend micro-simulation models to include behavioral relations is a topic of current research. This paper tries to survey a few issues of principles and current practice in this work.

2. Model structure, general issues.

Two uses and two different approaches to micro-simulation

The choice of general modeling approach depends on the intended uses of the model. A model, which will be used operatively for forecasting and policy recommendations, need be firmly based in an empirical reality and its relations should have been estimated from real data and carefully tested using well-established statistical and econometric methods. In this case the feasibility of an inference to a real world population or economic process is of great importance.

In research micro simulation can also be useful for other purposes, for instance, to explore the general consequences of alternative assumptions about the behavior of economic agents and their interaction in markets, but without ambition to draw an inference to a particular economy. In this type of application a micro simulation model is used very much in the same way as a conventional economic model in mathematical form. Given certain assumptions one wishes to explore their implications. The reason to use micro simulation rather than a more conventional analysis usually is that the model is highly nonlinear, it takes institutional rules and constraints into account and it includes the interaction of economic agents, which might be difficult to handle in a conventional mathematical analysis. These simulation models usually have a relatively weak empirical basis. Their relations are not always estimated by econometric methods and it might not even be possible to get the data needed for a statistical inference. The assignment of parameter values in these models are instead more or less ad hoc and their plausibility is checked by "calibrating" model predictions against a few observable key statistics. (For a discussion of this procedure see Klevmarken, 1980.) These models should thus be seen as a complement to a conventional economic analysis. In addition to a better understanding of how economic agents and markets might work under alternative institutional constraints, they might also give suggestions about new data, which need be collected in order to make a proper econometric analysis feasible. Examples of studies of this kind are Bergmann(1990), Ballot(1991), Eliasson(1991) and Wolfson(1996).

The boarder line between what we might call ”empirical models” and ”abstract models” is not always that clear. Most modelers use some kind of empirical information to determine parameter values but often without examining whether the model is identified, what properties the parameter estimates have and what kind of inference is permitted. These are circumstances sometimes driven by a desire on the part of the researcher to ”do something” without having the proper empirical bases for doing it! The micro simulation approach has therefore been discredited by the use of models with unrealistic assumptions based on data sets merged from a variety of sources and still used to produce statements about a real life economy. There is thus a need to structure the micro-simulation approach, clarify the inference problems and discuss when a model permits the use of different samples (data sets) to estimate subsets of parameters. This paper is an attempt in this direction and it thus focuses on the first type of micro-simulation applications, that is on empirical models rather than on abstract models.

A distributional representation of a MSM

In the micro simulation approach, the distributional properties of the economic variables are of key importance since these properties usually are our primary interest and not only a set of assumptions made for the convenience of estimation. For this reason it is natural to write a micro simulation model in distributional form. Assume that we distinguish between endogenous variables, i.e. variables explained by the model, and exogenous or cause variables, which explain the endogenous variables in the sense that they determine their distribution. Both the endogenous and the exogenous variables are stochastic variables, and the class of exogenous variables might include predetermined endogenous variables. In a very general form the model is,

$$f_{YX}(y, x \mid q) = f_{Y \mid X}(y \mid x, q_1) \cdot f_X(x \mid q_2), \qquad (1)$$

where Y is a vector of endogenous variables, X a vector of exogenous variables, and q, q_1 and q_2 parameter vectors. The dimensions of Y in general span the number of endogenous variables, the cross-sectional dimension of observational units and the time dimension. We thus assume a multivariate distribution f_{YX} and the conditional distribution for the endogenous variables given the exogenous. The distribution f_X is not explained by the model but exogenously given and it gives us the initial conditions for the simulation as well as any exogenous variables need during the simulation. $f_{Y|X}$ is the core of the simulation model which specifies how the exogenous X determine Y. X could for instance be pre-tax incomes, Y post-tax incomes and $f_{Y|X}$ the tax rules with parameters q_{1}, or $f_{Y|X}$ could be an economic model or a combination of an economic behavioral model and a set of legislative rules. Micro simulation aims at simulating the marginal distribution

$$f_Y(y) = \int_x f_{YX}(y, x \mid q)\, dx, \qquad (2)$$

or some statistics based on it.

In order to use the model to compute f_Y, i.e. without simulation, we would not only have to know $f_{Y|X}\left(y \mid x, \hat{q}_1\right)$, where $\hat{q}_1$ is an estimate of q_1, but also the distribution of the exogenous variables f_X. In general, there is little theory which could be used to specify f_X since , by definition, the X-variables are exogenous. The micro simulation approach circumvents this difficulty by simulating the model with a sample from f_X. For a sample of, for instance, individuals, household, or firms, the observed x-values are used to simulate the corresponding y-values. If this sample is a random sample from f_X, it is possible to use the simulated y-values for inference about f_Y without any assumptions about f_X. One might view micro-simulation as a way of replacing f_X by the corresponding empirical distribution.

<u>Structuring a large model</u>

Modeling a big micro model with many variables is a difficult task, and it is usually not practical or feasible to specify $f_{Y|X}$ in one step. Usually we attempt simplifying assumptions, which allow us to work with marginal distributions. How this is done will have implications for data need, estimation and simulation. Assume, for instance, that the vector Y can be partitioned into two independent subvectors Y_1 and Y_2, i.e.

$$f_{Y|X}(y|\, x, q_1) = f_{Y_1 \mid X_1}(y_1 \mid x_1, q_{11}) \cdot f_{Y_2 \mid X_2}(y_2 \mid x_2, q_{12}), \qquad (12)$$

where the vectors X_1 and X_2 are either identical with X or subvectors of X. They may or may not have variables in common. This factorization of the model facilitates estimation and testing. To estimate

$f_{Y_1 \mid X_1}$ we only need a sample of $(y_1 x_1)$ -observations and to estimate $f_{Y_2 \mid X_2}$ we could use a different sample of (y_2, x_2) -observations. No sample including all endogenous and exogenous variables is thus needed to estimate the model. If X_1 and X_2 have no variable in common, *and* if X_1 and X_2 are stochastically independent, then

$$f_{YX}(y, x \mid q) = f_{Y_1 \mid X_1}(y_1 \mid x_1, q_{11}) \cdot f_{X_1}(x_1) \cdot f_{Y_2 \mid X_2}(y_2 \mid x_2, q_{12}) \cdot f_{X_2}(x_2). \qquad (13)$$

It is then possible to simulate each part of the model separately and no sample needs to include all exogenous variables. If, however, X_1 and X_2 are not independent, the simulations must be done with the full model, although each submodel can be estimated separately.

Economic theory sometimes suggests that the assumption of independence in eq. (12) is unrealistic, but might suggest another partition.

$$f_{Y|X}(y, x|q_1) = f_{Y_1|Y_2 X_1}(y_1|y_{2,} x_1; q_{ll}) \cdot f_{Y_2|X_2}(y_2|x_2; q_{12}) \qquad (14)$$

One could, for instance, think of $f_{Y_2 \mid X_2}$ as the distribution of wage rates conditional upon age, schooling and other exogenous variables and $f_{Y_1 | Y_2, X_1}$ as the distribution of hours of work conditional on the wage rate and exogenous variables. Another example is to let $f_{Y_2 \mid X_2}$ be the distribution of the husband's workhours and $f_{Y_1 | Y_2, X_1}$ the distribution of the wife's hours. In this case one will need data which include both Y_1 and Y_2 and when estimating θ_{11} one would have to use methods which take account of the endogeneity of Y_2 . The model could however, be simulated recursively. Given the X_2 values Y_2 is first simulated and the result is jointly with the X_1-values used as input to simulate Y_1. This procedure does not imply any assumption about a recursive structure. $f_{Y|X}$ could in principle be partitioned in many different ways. One could, for instance, reverse Y_1 and Y_2 and define $f_{Y_1 | X_1,}$ and $f_{Y_2 | Y_1, X_2}$. From a simulation point of view any partition could do, but economic theory might suggest one partition rather than another which would facilitate interpretation and validation of the corresponding parameter estimates.

Let us factorize the joint distribution of eq. (1) by time period,

$$f_{YX}(y,x;\theta) = f_{Y_T|Y_{T-1}\cdots Y_1,X_T}(y_T|y_{T-1},\cdots,y_1,x_T;\theta_{IT})\cdots f_{y_t|y_{t-1}\cdots y_1,x_t}(y_t|y_{t-1},\cdots,y_1,x_t;\theta_{it})$$
$$\cdots f_{Y_1|X_1}(y_1|x_1;\theta_{11}) f_{X_1\cdots X_T}(x_1,\cdots,x_T); \tag{15}$$

where t=1,..., T indexes period. In practical work we usually only estimate models with lags of at most a few periods (τ) and we usually assume parameter stability, $\theta_{1s} = \theta_{1t} \quad \forall s,t$. The typical conditional distribution in the above expression can then be simplified to

$$f_{Y_t|Y_{t-1}\cdots Y_{t-\tau}}(y_t|y_{t-1,\cdots,}y_{t-\tau},x_t;\theta_1.) \tag{16}$$

To estimate such a model one will in general need panel data with at least τ observations on each individual. When the model is simulated it will not only give estimates of cross-sectional distributions $f_{Y_t}(y_t)$ but also of the joint distributions $f_{Y_t\cdots Y_{t-\tau}}(y_t\ldots y_{t-\tau})$, which implies that it is possible to simulate both "mobility" , for instance earnings mobility, and individual life cycle paths.

Such a model is, however, very demanding in terms of data, estimation and handling. Suppose one is only interested in good estimates of the cross-sectional distributions $f_{Y_t}(y_t)$ but not in mobility, then one could try to estimate the marginal distribution $f_{Y_t|X_{1\ldots}X_t}$ directly. This is an important but sometimes overlooked point. Cross-sectional (static) models might give good representations of cross-sectional distributions but they are likely to create excessive individual mobility when simulated. For instance, a cross-sectionally estimated earning function might produce a good image of the cross-sectional distribution of earnings, but because it lacks memory it will not produce good estimates of individual earning paths but exaggerate earnings mobility.[2] Similarly a conventional

[2] Estimates of earnings functions from panel data have demonstrated a positive correlation between succesive earnings observations. See, for instance, Hart(1976, 1980), Creedy, Hart & Klevmarken(1980), Hause (1977,1980), Lillard & Willis(1978), Klevmarken(1993)

static model of labor supply of the Hausman type might give good estimates of the cross-sectional distribution of desired work hours but it will most likely exaggerate hours mobility.[3]

If one is only interested in the cross-sectional distributions the mobility issue might not be very important. One could even argue that (minor) specification errors in a dynamic model might create a drift in the cross-sectional distributions which could be avoided with a static model. But if issues of dynamics and mobility are important in the applications of the micro simulation model one can clearly not use cross-sectional models. They should for instance be inadequate for an inference about the speed of adjustment to policy changes. If the model includes accumulation processes such as those of savings and wealth, overestimated income mobility is likely to lead to underestimated wealth dispersion. From a statistical point of view it would also be inefficient not to use the available information of past realizations to simulate (forecast) the future. Ideally one should of course be able to design dynamic models which are able to predict well both cross-sectional distributions and mobility.

One possible partitioning of eq. (15) is to do it by individual, i. e. to assume that the behavior of one individual is independent of any other individual. This approach permits the simulation of the entire path for each single individual without using information about the others. The model is rerun once for each individual and the number of runs equals the sample size. An example of a model within this approach is HARDING[4]. An advantage of this approach is that it might not be necessary to store and retrieve large amounts of intermediate simulation results. In this case it is also relatively simple to use models in continuous time like event history models. An obvious disadvantage is that this approach makes it impossible to take advantage of one of the main attractions of micro simulation namely the modeling of interactions between economic agents. In this approach the simulated outcome for individual i will in no way influence the outcome for j. With a dynamic model (in the micro simulation sense) which simulates demographic events like births, marriages, separations and deaths this approach would seem less useful. Also when the interaction of sellers and buyers in markets are explicitly modeled one would have to use another approach. In practice it

[3] The distribution of actual hours of work is usually not well simulated by a Hausman-type model. The simulated distribution does not have the peaks at full-time and half-time hours usually observed in data.
[4] Harding (1990, 1993)

is, however, not feasible to allow the behavior of everyone to depend on that of everyone else. Inter individual dependence might be limited to certain sub models or to more narrowly defined groups of individuals such as members of a household. In the simulation process one could then take advantage of this partial independence.

The alternative is to simulate the outcome for all individuals time period by time period. Depending on the model structure it might then become necessary to store large amounts of intermediate simulation results. For instance, if the model simulates transfers between generations in the form of inheritance, information about kinship has to be stored and one might have to keep track of personal property as distinguished from joint property in a marriage or consensual union.

Depending on the model structure one might also choose a particular simulation order within a unit time period. As indicated above the model might have a hierarchical structure, which permits simulation in a given module sequence.

Independently of approach it is in practice necessary to impose a structure on the relations between different types of economic actions. It is impossible to estimate a model in which decisions about schooling, family size, work, housing etc. are jointly determined. The particular structure chosen will depend on the general purpose of the model and also on data availability. For instance, if no data source has the information required to estimate models for both housing and work hours it is impossible to estimate a model which determines housing and labor supply jointly. An approach taken in a few models, for instance MICROHUS[5] and NEDYMAS[6], is to a priori assume a hierarchical structure between major model modules. This could, for instance, imply that demographic changes and decisions about household formation are assumed to precede decisions about market work. Thus, only past but not current decisions about market work will then influence decisions about household formation, while current decisions about household formation, for instance to have a child, will influence market work. As pointed out above this hierarchical structure is not necessarily recursive, which would imply certain assumptions about the correlation structure of the model. If it is, it has primarily implications for estimation, for instance, if having a new child

[5] Klevmarken et.al.(1992), Klevmarken & Olovsson (1993)

should be treated as exogenous or endogenous to the labor supply decisions. When the model is simulated it is possible in principle to follow the hierarchy whether the model is recursive or not.
In practice the structure of a model might not be specified such that it is easy or even analytically feasible to see the correspondence between the structure and the distributional representation of the model. This will in particular be the case if the processes of a model structure are independent. Experience with MICROHUS, for instance, shows that it is difficult to maintain a hierarchical simulation structure because results from a process later in the hierarchy are needed in a process in the beginning of the hierarchy. For instance, decisions about having a child depend on the woman's wage rate net of marginal tax but a marginal tax rate cannot be computed until housing and labor supply decisions have been simulated. Any model which assumes optimization within a budget set will have this problem, and it might not be feasible analytically to transform the structural representation into a distributional representation. A time-consuming alternative is to simulate all processes iteratively for each individual until stable individual solutions are obtained.

In summary we thus find that the structure of a model will determine data need, and estimation and simulation procedures. Assumptions about independent processes make possible the use of independent samples for estimation purposes. They could also justify statistical matching of separate samples to form the basic household population on which all simulations are based. In a model with discrete time there are in principle six alternative ways to choose s simulation order depending on which of the three attributes "time-period", "process" and "individual" define the outer most, middle, and inner most simulation loop. If time is the inner loop then all time periods are first simulated for every individual and process, while if individual defines the inner loop, all individuals are simulated for every time period and process. The model structure might suggest which alternative is most convenient.

3. Behavioral modeling

A brief survey of behavioral modeling in MSM

[6] Nelissen(1994)

Since the start of micro-simulation in economics with Orcutt's seminal 1957 article (Orcutt, 1957) and the first dynamic micro-simulation model in the United States (Orcutt et al 1961) this approach has in the past almost 40 years been both successful and met with a great degree of skepticism. Successful to the extent that static micro-simulation models have become a standard tool for policy evaluation in most Western governments, but at the same time less accepted among academic economists, who sometimes find unacceptable the compromises between theoretical and methodological rigor and what is feasible given insufficient data and resources. They have not seen MSM as a useful tool in developing and testing theory.

None-the-less there is a rather impressive list of MSM as shown by recent surveys, for instance, Merz (1991), the OECD Committee on Fiscal Affairs (1988), Mot (1992), Sutherland (1995) and Galler(1997), and by a number of conference volumes, for instance, Bergmann et al (1980), Orcutt et.al. (1986) and Harding (1996). Many MSM are static without behavioral relations and this is in particular true for models run by government agencies, international organizations and consulting firms. Behavioral modeling is still to a large extent an academic exercise. Tables 1-3 list models, which at least include some behavioral relation. These tables are not exhaustive but give a sample of more or less well-known models. What defines a behavioral relation is not crystal clear. A matrix of transition probabilities differentiated by age and sex is a simple behavioral relation in the sense that behavior is differentiated by age and sex. Some models include relations derived from economic theory but this is not a requirement for a model to become classified as behavioral. For instance, demographic models of transition matrix type have been included. Although these models include behavioral heterogeneity in a broad sense many of them do not capture any behavioral response to policy changes. Their behavioral relations do not include the relevant policy parameters.

There is also another type of behavioral modeling applied to micro simulation models where behavior is modeled at an aggregate level. The aggregate implications are then subsequently disaggregated in a micro simulation model. An example is Meagher(1996). A dynamic general equilibrium model of the Australian economy is used to compute growth rates in the (factor) incomes of selected groups. These income changes are then fed into a static micro simulation model, which produces simulations of the after tax income distribution. A similar application is also given in Baekkgaard and Robinson(1997). In the following we will not pursue further the linkage with the macro economy.

Table 1 includes static MSM with behavioral relations. One may note that among the most common behavioral relations are those of labor supply but there are also models which, for instance, include expenditure functions to simulate the effects and changes in indirect taxation. The early models were designed in the United States while the Europeans caught up in the 1980s and now seem to dominate the work with static behavioral models.

Table 2 lists general dynamic models with behavioral relations. "General" here means two things. The model is not specialized for a very limited purpose or limited to a small group of individuals or households, and it contains more than a single or just a few behavioral relations. Most of these models are large and cover the whole household sector in a country and include modules which age their populations as well as modules which are more central to the general purpose of the models, for instance, labor supply relations which capture behavioral adjustments in the labor market to tax changes. The behavioral relations of all models, however, are not specified such that behavior directly depends on the policy instruments, some are more of a "demographic" type. Also in dynamic modeling the Americans were pioneers. In Europe German scientists would seem to have worked relatively early with dynamic MSM.

Table 3 shows MSM with a more specialized aim and limited scope. Labor market behavior is dominating among these models as well but here is a greater variety of coverage: consumption behavior, housing demand, demand for energy, child care, telephone services and non-market time. Most of these models are probably more closely based on economic models and econometric testing and estimation than the big general dynamic models.

Behavioral modeling for three purposes

A Behavioral model can serve at least three purposes in a MSM. First it could be used to impute missing data as an alternative to statistical matching (see Klevmarken, 1983).[7] Suppose there are X_1 data in the data set used for simulation, but X_2 data are missing, and that there is another data set with both X_1 and X_2 data. If X_2 can be related to X_1 by way of a behavioral model then the model can be estimated from the second, external data set and used in the micro simulation model to simulate X_2 . Such a relation need not be based on theory about behavior, what is needed is a good predictive relation, but if it is delivered from good theory one would probably have more confidence in its predictive ability.

Behavioral models can also be used to age the simulation population (sample). This is usually done by introducing mortality tables, relations for the birth of new individuals, marriages, separations etc. Similarly behavioral relations could update other characteristics of the population like labor force participation, unemployment, hours of work, wage rates, housing, child care etc. As long as the purpose is limited to updating behavioral relations are only needed to the extent that they yield more stable and precise predictive relations compared to alternative ways of updating the population. In practice many models use matrices of transition probabilities estimated separately for a few subgroups of the population, for instance, by age and sex, but with no strong connection to theory.

The third and perhaps most interesting application of behavioral relations is to capture behavioral adjustments to policy changes. A necessary requirement of a behavioral model to satisfy this purpose is that the policy parameters directly or indirectly enter the model. This is normally not the case in the simple transition matrixes used for aging and updating. For instance, in a study of the distributional effects of income tax changes the labor supply function should be such that labor supply depends on the tax rates (and virtual income). A second requirement is that the behavioral relation is stable such that its parameters do not change as a result of policy changes. This is an issue much discussed, for instance, in relation to the recent major tax reforms in many countries. Can labor supply relations estimated on data collected before the reforms be used to predict or evaluate the effects of the reforms? Are the parameter estimates stable in spite of the large tax changes in some countries? The same kind of concern could be raised when MSM are used to simulate

[7] A similar application is found in Birkin and Clarke(1989). They use proportional fitting in a micro-simulation approach

processes of long duration, for instance changes in pension systems. Is it possible to extrapolate long into the future earnings and labor supply relations estimated from short time spans of data?

Behavioral modeling in static micro simulation models.

To emphasize the policy evaluation application of micro simulation models and make behavioral adjustments explicit assume there are three kinds of variables: Policy variables X_p, target variables Y and all other variables X_{np}. For instance, one could think of X_p as tax rates and tax bases, Y as taxes paid and after tax income, while X_{np} would include variables needed to compute taxes like labor incomes and nonlabor incomes, and group indices like sex, marital status, nationality, region, etc. In a conventional nonbehavioral static tax-benefit model policy variables are related to the target variables through the tax and benefit system conditional on X_{np}. For a single individual we could write this relation as,

$$Y=T(X_p, X_{np}) \tag{17}$$

In a micro-simulation application of this model we compare the distributions of

$$Y_1=T(X_{p1}, X_{np\,0}) \tag{18a}$$

and

$$Y_2=T(X_{p2}, X_{np\,0}). \tag{18b}$$

for two different policy regimes X_{p1} and X_{p2} and a given set of population characteristics $X_{np\,0}$.

Replicated static micro-simulation actually approximates the distribution,

$$f(Y_1, Y_2, | X_{p1}, X_{p2}, X_{np\,0}). \tag{19}$$

from which we can compute the marginal distributions by simple summation,

to estimate small area income distributions.

$$f(Y_1| X_{p1}) = \int f(Y_1| X_{p1}, X_{np\,0})f(X_{np\,0})dX_{np\,0} \qquad (20a)$$

and

$$f(Y_2 | X_{p2}) = \int f(Y_2| X_{p2}, X_{np\,0})f(X_{np\,0})\, dX_{np\,0} \qquad (20b)$$

and various distributions conditional on subsets of $X_{np\,0}$, for instance on gender, type of family, region, etc. It is here assumed that the empirical distribution of $X_{np\,0}$ in the micro-simulation model closely approximates the true distribution of $X_{np\,0}$, which for instance would be the case if the sample used for micro simulation is a simple random sample from the target population. (If the sample is drawn with unequal sampling probabilities one would have to compensate for this by using appropriate sampling weights.)

If the model T(..) is just a nonstochastic tax-benefit model the distribution (19) could be a degenerate one-point distribution. It is of course still possible to compute the marginal distributions (20a) and (20b) and various conditional distributions. If T(..) is a stochastic model and the simulation is only done once we might not get a good approximation of the distribution (19), depending on how frequently $X_{np\,0}$ is replicated in the simulated population. The reason is of course that we will only get one observation (Y_1, Y_2) for each individual. However, we can still compute good approximations of the marginal distributions (20a) and (20b) and various interesting conditional distributions.

The marginal distribution $f(Y_1, Y_2| X_{p1}, X_{p2})$ is of particular interest, because it tells us, for instance, about the after tax income mobility. What share of the population move from one after tax income decile to another as a result of the change in policy? Although $X_{np\,0}$ has been integrated out $f(Y_1, Y_2| X_{p1}, X_{p2})$ is only valid for a population with the characteristics $X_{np\,0}$. Please also note that nothing is said and nothing can be said about the individual trajectories through time which result from a policy change. Nor do we say anything about changes through time in the distribution of Y or when a certain share of the population has moved from one decil to another.

Assume now that in addition to the tax-benefit model some of the variables X_{np} also depend on the policy regime. To mark their changed status call them Y_p , and keep the old notation X_{np} for variables which are truly exogenous. A static model with behavioral adjustments could be written as,

$$Y=T(X_p, Y_p, X_{np}) \tag{21a}$$

$$Y_p=C(X_p , X_{np}) \tag{21b}$$

Where C is a function which relates the policy variables to individual behavior of relevance for the target variables. One could, for instance think of C as a labor supply model which determines hours of work (Y_p) as a function of the tax rates, deductions and thresholds (X_p) and wage rates and nonlabor incomes (X_{np}). All these variables thus jointly determine disposable income.

Micro-simulation of this model will, for instance, involve a comparison of the following two distributions,

$$f(Y_1, Y_{p1}| X_{p1} , X_{np0}) \text{ and } f(Y_2, Y_{p2}| X_{p2} , X_{np0}) \tag{22}$$

and the corresponding pairs of marginal distributions,

$$f(Y_1| X_{p1} , X_{np0}) \text{ and } f(Y_2 | X_{p2} , X_{np0}) \tag{23}$$

and

$$f(Y_1| X_{p1}) \text{ and } f(Y_2 | X_{p2}) \tag{24}$$

Again there is no time dimension in this model. Although it is possible to compute the distribution $f(Y_1, Y_2, Y_{p1}, Y_{p2} | X_{p1}, X_{p2})$, which, for instance could tell us what share of the unemployed became employed as a result of the policy change, it does not tell us when. Depending on how the model is designed and estimated and the simulations done this distribution might also vastly overestimate mobility. If C(..) is a stochastic model such that the implicit individual random error is drawn independently for each policy regime then the model neglects any unobserved individual heterogeneity and it would simulate too much mobility. Although such a model could not be used to evaluate the ”true”distribution $f(Y_1, Y_2, Y_{p1}, Y_{p2} \mid X_{p1}, X_{p2})$ it might still simulate well the marginal distributions (22), (23) and (24).

Suppose, for instance, that C is a static labor supply model of the Hausman type and Y_p is hours of work, and that this model is simulated for two different tax regimes. If the random “optimization errors” of the Hausman model are IID, and independent sets of errors are drawn for the two tax regimes, then mobility in hours will most certainly become exagerated. This excess mobility will then transmit to disposable income. In this example the simulated joint distribution of disposable income and hours of work for the two policy regimes $F(Y_1, Y_2, Y_{p1}, Y_{p2}| X_{p1}, X_{p2})$ is the product of the marginal distributions $f(Y_t, Y_{pt}| X_{pt})$, t=1,2. Although this is believed to be unrealistic it does not exclude that each of the simulated marginal distributions are good approximations.

One way to reduce mobility is to use the same seed when the two sets of “optimizing errors” are drawn. Each individual will then have the same error in both tax regimes. However, there is no guarantee this approach will give a realistic representation of mobility. It might well create too little mobility. The simulation procedure should be based on empirical studies of mobility, and then a static model is not a good framework.

An alternative explanation to a smaller mobility compared to a purely random process is the presence of state dependence.[8] Assume, for instance, that,

$$Y_p=C(X_p, X_{p0}, Y_0, Y_{p0}, X_{np}). \tag{25}$$

In this model behavior does not only depend on the policy X_p chosen and the exogenous characteristics X_{np}, but also on a reference "level" of policy variables X_{p0}, target variables Y_0 and behavioral response variables Y_{p0}. For instance, the effect of a tax change on labor supply might depend the level of unemployment when the tax change is implemented. It might also depend on the

[8] In empirical work it might not always be easy to distinguish state dependence from individual heterogeneity.

nature of the tax system used immediately before the taxe change. The behavioral response to a change in the marginal tax rate may depend on whether the rate was higher or lower prior to the change.

Families of behavioral models

Only human imagination, data availability and computer resources limit the structures and forms behavioral models could take, and it is certainly possible to group existing models in many different ways. The following classification indicates the variety of approaches and functional forms used, many of which can usually be found within one and the same micro simulation model.

1. Models of transitions between different states.
   To this class belong models of transition probabilities like Markov-models, probit, logit, multinomial logit and ordered probit models to mention a few. It also includes event history (hazard rate) models.

2. Count data models.
   Count data models like for instance Poisson regression have been used to model the number of occurrences of an event in an a priori specified time span or the number of time periods an individual belongs to a certain state, for instance, the number of months of unemployment in a year or the number of weeks reported absent from work due to sickness in a year. These models have been used when event history data were not available, one only knew for how many weeks a person had been in a state, for instance sick, but not if these weeks formed one or more spells of sickness.

3. Continuous data models
   To this group belong conventional linear and nonlinear regression models, equation systems etc. In micro simulation models for earnings functions, models for work hours and expenditure

functions are examples of this model type.

4. Random assignment schemes (statistical matching)
   The models of the first three classes above belong to the conventional econometric paradigm of estimating an average structure from which there are only random deviations. In the first two cases one estimates (average) probabilities conditional on certain individual characteristics and the deviation from the most probable outcome is accomplished by chance, by throwing a ”loaded dice”. In the third case we simulate deviations from the average by adding random disturbances to the ”systematic” part of the model. In random assignment schemes like statistical matching, the model structure is implied and never estimated. It is only defined by the variables which define ”closeness”. The idea is to find a donor of data among the observations in the population which in some sense is similar or close to the receiving unit. Suppose for instance, that the original data set includes observations on income for two consecutive years for each individual. A simulated income distribution for a third year could be obtained by defining closeness between the donor’s income in the first year and the receiver’s income in the second year perhaps also between other variables like age and sex and then randomly select a donor among those who have the (approximately) same age, sex and income as the receiver. The donor’s income for the second year is then used as a prediction of the receiver’s income in year three. The implicit model assumption is of course that income transitions remain unchanged. With a similar but somewhat more elaborate approach Hussenius & Selén (1994) linked short panels of income data to life-cycle income paths to analyze how like-cycle incomes were influenced by tax and transfer changes. In the MICROHUS model (Klevmarken et.al. 1992, Klevmarken & Olovsson, 1994) the technique was used to simulate the properties of the house a family was predicted to buy (size, tax assessed value, size of mortgage and interest paid).

   Advantages with the random assignment technique are thus that no assumptions of functional forms or distribution families are needed, if preserves the variation and (most of) the correlation already present in the original data, and it is nonparametric so there is no estimation of unknown parameters. The choice of variables and the measure used to define closeness can be tested by goodness of fit. A disadvantage though is that the statistical properties of the resulting

”predictions” are incompletely known. Results from the imputation literature might be relevant. Another practical disadvantage is that the random assignment technique can never predict beyond the range of values already present in the original sample.

With exception of the random assignment technique the more traditional approaches to modeling invites the model builder to become excessively parsimonious with the number of estimated parameters and the models thus tend to become of the type average behavior with random variation. The possibility to permit people to behave fundamentally differently and to study the interaction between people with different aims which is feasible within the micro simulation approach, is usually not taken advantage of. For instance, with panel data, only a few observations for each individual are needed to estimate a utility function for each individual and allow everyone to maximize her own utility. One could also think of models when some individuals maximize their utility while the behavior of others are guided by something else than utility maximization!

The time unit in dynamic models

Continuous time models have the attraction to accommodate any time span one might like to use and also to permit different time spans for diffent purposes and in diffent submodels. However, a continuous time model which would permit the complexity and interaction between individuals which is necessary in most micro simulation models would become exceedingly difficult to estimate and simulate. It is thus probably not practical to have the entire MSM formulated in continuous time. Continuous time might, however, be useful in certain sub models.

In micro simulation models, which include income tax systems it is necessary to use the time unit of a year because income taxes are usually assessed annually. Some benefit systems, however, operate on shorter time spans. For instance, compensation for sickness and unemployment and social assistance might be given on a daily, weekly or monthly basis for the duration of the particular state. There is thus a need to use more than one time unit within a micro simulation model. This could be accomplished in several ways. One approach is to run simulation loops within a year for those submodels which operate on a shorter time unit, another approach is to use count models which simulate the number of days, weeks or months a person is sick, unemployed, etc in a year. A third approach is to use a continuous time model, for instance an event history model, to simulate the date when a person enters and leaves a certain state. The last approach has the advantage that it can accommodate a particular problem which sometimes occurs, namely that benefit rules are changed such that the change takes effect at a date other than the 1st of January. Galler(1996) discussed the relative advantages and disadvantages of models in discrete time and continuous time. His main conclusion was that a discrete time framework with comparatively short time periods appears to be best-suited causal modeling in dynamic micro-simulation models.

4. Data and inference

Micro simulation demands much data, both for estimation of behavioral relations and for the model population of simulation units (individuals, households, firms). As explained above, if it is possible to factorize the model structure in a convenient way one might not need one single big sample but could do with several separate samples. For instance, if decisions about housing are assumed independent of decisions about market work, then it might be feasible to use one sample to estimate the demand for housing and tenure choice and another sample to estimate earnings functions and labor supply functions. However, one single sample avoids many problems and obscurities encountered when data are collected from more than one source. It is usually doubtful whether all samples can be considered drawn from one and the same population and there are problems with differences in the definition of units of observations and in variable definitions. With one single sample it is also possible to test assumptions about conditional independence. When limited data availability dictate what model assumptions are needed to justify the use of more than one sample these assumptions cannot be tested.

If we disregard the problems mentioned above when more than one sample is used, a micro simulation model can in principle be used in two different kinds of inference, either an inference to a finite population or an inference to the ”superpopulation” or data generating process of which the micro simulation model is a mirror image. In both cases the selection probabilities of the sample used in the simulations must in general be used to weight the simulated Y-values to obtain a consistent estimate of the distribution of Y, f_Y in eq. (2).

Depending on the sampling design and the model structure $f_{Y|X}(y|x, \theta_1)$, the sampling weights should or should not be used when the model parameters are estimated. If the sample selection does not depend stochastically on any of the endogenous variables, we can estimate q_1 — but in general not simulate Y — as if the sample was obtained by simple random sampling. If we resort to ML-estimation, this result follows from the structure of the likelihood function. The likelihood of a sample of one observation is

$$L(q_1) = \frac{f_{Y|X}(y|x, q_1) \cdot f_X(x) \cdot P(s|x)}{\int\int f_{Y|X}(y|x, q_1) \cdot f_X(x) \cdot P(s|x)\, dydx} = \frac{f_{Y|X}(y|x, q_1) \cdot f_X(x) \cdot P(s|x)}{\int f_X(x) \cdot P(s|x)\, dx}, \qquad (26)$$

where $P(s \mid x)$ is the selection probability given x. Since P and f_X do not depend on q_1, the likelihood function for a sample of n units will take its maximum for the θ_1 - value which maximizes

$$\prod_{i=1}^{n} f_{Y|X}(y_i \mid x_i, q_1) \cdot \qquad (27)$$

However, if P depends on Y the maximum of the likelihood function will depend on P. (See for instance, Rubin(19776), Manski & McFadden(1981), Little(1982) and Hoem(1985).)

Sampling theory will only permit an inference to the population from which the sample was drawn. An inference to the population of, say, another year than the year which generated the sample of the model population, is in general not possible. What we might do, in particular with a static model, is to predict the consequences of new values of the exogenous variables for the original population. Any inference to the true population of another year than the one generating the sample would have to be based on good faith. For instance, a static tax-benefit model could answer the question what is the effect of a given policy change on the income inequality of the population from which the model sample was drawn. It could also answer the hypothetical question what this effect would have been had the income distribution of the population been another than that actually observed. One could also compare the policy outcome for two different populations if there is a random sample from each of them. In practice this is sometimes accomplished by ”reweighting” the original sample using external information on demographic distributions, but it is usually doubtful whether the reweighted sample can be used for a proper inference to a real life population.

In the case of a dynamic model the situation is even more complicated because the model will simulate the birth of new individuals and households and the disappearance of old, and these new units will have no sampling weights. If a child is born by a single mother and the model is such that no characteristics of the father determine the probability of a new child, then the mother's sampling weight could be applied to the child. But if the child is the result of a marriage between two persons with unequal sampling probabilities, what sampling weight should then the child be given?

It is obvious that a self-weighted sampling design would facilitate both estimation and simulations considerably. If the sample is not self-weighted the problems in the simulations phase with unequal sampling weights could be avoided if the original sample is expanded such that a number of "copies" are made of each sample member proportional to its sampling weight. One could also perform a random selection from the original sample with replacement and with selection probabilities proportional to the sampling weights.

With a dynamic model it becomes, however, rather pointless to maintain the fiction of an inference to the original population because the purpose of the whole simulation exercise is to simulate changes in the population. An inference to real world populations depends entirely on the ability of the simulation model to capture changes in the population in a realistic manner. One would have to abandon the idea of an inference from a sample to a finite population and take the conventional econometric view, i e the micro simulation model is a representation of the data generating process. The model can be tested against data using conventional econometric methods and if it passes the tests, predictions can be made and their stochastic properties evaluated.

Estimation using external information

Given the complexity and mixture of model types and functional forms in a large MSM its parameters are usually estimated in a piecemeal way, submodel by submodel. As explained above the model structure might justify such a procedure, but in most cases this is probably done just for convenience. It implies that no model-wide estimation criterion is used, and given that such a criterion exists that the estimated parameters most likely are not optimal.[9]

It is not at all obvious how one would choose a model wide estimation criterion. For instance, how should the simulation errors in one variable (time period) be weighted against errors in another? It is conceivable that these weights might depend on the application of the model. In one case it might be

[9] Hooimeijer(1996) page 45 notes that " A problem with micro-simulation models is the internal inconsistencies that occur when various parts are put together. These inconsistencies arise from unobserved restrictions on partial behaviour … . this indicates a major advantage and a major drawback of the method. The advantage is that this shows that the whole is more than the sum of its parts. The drawback is that no elegant solution to this problem has been offered."

more important to simulate well hours of work while in another tenure choice might be more important. In a general purpose model it is, however, hardly feasible to reestimate the model for each application, one would prefer a criterion which works well in most cases.

To give an example of the implication of a model-wide estimation criterion compared to a piece meal approach consider the following simple two equation model:

$$y_{1t} = \beta_1 x_t + \varepsilon_{1t}; \qquad E(\varepsilon_i \varepsilon_j) = \begin{cases} \sigma_1^2 & \text{if } i=j=1. \\ \sigma_2^2 & \text{if } i=j=2. \\ 0 & \text{if } i \neq j. \end{cases} \qquad y_{2t} = \beta_2 y_{1t} + \varepsilon_{2t}; \tag{28}$$

It is welknown that OLS applied to each equation separately will give consistent estimates of β_1 and β_2. The estimate of β_1 gives the BLUP $\hat{y}_1 = \hat{b}_1 x_t$ while predictions of y_2 outside the sample range is $\hat{b}_2 \hat{y}_1$. However, this suggests the following model-wide criterion,

$$\frac{1}{s_1^2} \sum_t (y_{1t} - \hat{y}_{1t})^2 + \frac{1}{s_2^2} \sum_t (y_{2t} - \hat{b}_2 \hat{y}_{1t})^2; \tag{29}$$

Minimizing this criterion with respect to $\hat{b}_1$ and $\hat{b}_2$ yields the OLS estimator for β_1 but the following estimator for β_2,

$$\hat{b} = \frac{\sum_t y_{2t} x_t}{\sum_t y_{1t} x_t}; \tag{30}$$

In this case both the ”piece meal” OLS estimator of β_2 and the ”system-wide” instrumental variable estimator are consistent but the OLS estimator is not optimal if minimum prediction errors are aimed at.

Simulations in a micro-simulation model are, however, usually not obtained in the same way as the predictions above. They would underestimate the dispersion of y_1 and y_2. In practice we usually add an error term, say $\tilde{e}$, to the prediction, for instance

$$\tilde{y}_{1t} = \hat{b}_1 x_t + \tilde{e}_t; \tag{31}$$

in order to preserve the variance of y. Considering the model specification it might seem natural to add an independent error with variance σ^2 (or in practice $\hat{s}^2$). This would, however, exaggerate the variance of y, because

$$\mathrm{Var}(\tilde{y}_{1t}) = \mathrm{Var}(\hat{b}_1 x_t) + \mathrm{Var}(\tilde{e}_1) = \sigma^2\left(\frac{x_t^2}{\sum x_t^2} + 1\right) > \sigma^2; \tag{32}$$

to obtain a simulated distribution with the same variance as that of y the random error added to the BLUP should have the variance

$$\tilde{s}^2 = \sigma^2\left(1 - \frac{x_t^2}{\sum x_t^2}\right). \tag{33}$$

In practice one would have to estimate σ^2 by the residual variance.

In micro-simulation one thus trades a predictor which minimizes the sum of squared prediction errors for a predictor with a larger variance in order to simulate well the variance of the distribution of y. This is another way of saying that the objective function is not really the sum of squared prediction errors, but it also involves the variance of the simulated distribution or more generally all the properties of the distribution of interest to the analyst. In micro-simulation we do not only focus on the means!

In a large simulation model with many relations and distributions to simulate the choice of a model-wide estimation criterion becomes intricate. How should one weight different properties of simulated distributions against each other, and how should one compare simulation errors in one variable to those in another? A natural criterion function to consider is the likelihood function, but for a large model it is probably impossible to give an analytic expression for the joint likelihood function. However, the model can be used to simulate the joint distribution for a given set of parameters and initial condition. The model structure and simulation routines implicitly define the distributional properties of the model. The simulated distribution can be used to compute an approximation of the likelihood of the observed sample. By repeating these computations for alternative parameter values it might be possible to get simulation-based maximum likelihood estimates. There are of course numerical and statistical problems which have to be analyzed.[10]

In practice some model builders have followed a different approach, namely to align the model to external bench mark data. Population totals and means from official statistics or estimates from surveys not used to estimate the model are sometimes used as bench marks. If a model is to gain credibility with users they often require that the model is able to reproduce the basic demographic structure of the population and predict well-known bench marks like for instance, the labor force participation rate, the unemployment rate, the mean and dispersion of disposable income, etc. For this reason model builders have forced their models to predict these numbers without error. In CORESIM, for instance, this alignment is done by adjusting the simulated values (and not the parameter estimates).

A natural way to incorporate this kind of externally given information is to look upon the estimation problem as one of constrained estimation. Assume the micro-simulation model can be written in the following way.

$$Y_t = g(Y_{t-1}, X_t, \varepsilon_t, \theta) \tag{34}$$

[10] Simulated maximum likelihood estimates have been used for models of discrete choice and limited dependent variables to overcome the curse of dimensionality in these models, see for instance, Lerman & Manski(1981), Hajivassiliou and Ruud(1994), and Weeks(1993, 1997).

where $Y_t = \{y_{ikt}\}_{n\times K}$ is a matrix of *K* current endogenous variables for *n* individuals. $X_t = \{x_{ilt}\}_{n\times L}$ a matrix of exogenous variables, $\varepsilon_t = \{\varepsilon_{imt}\}_{n\times M}$ a matrix of random errors with expectation zero and some variance-covariance matrix Ω, and θ a vector of *P* parameters. Assume also that a sample of *Y* and *X*-variables is available for *n* individuals in *T* time periods.

Given some estimate of θ say $\hat{\theta}$, it is possible to define the following predictions within the sample period,

$$\tilde{Y}_1 = g(Y_0, X_{1,} \tilde{\varepsilon}_1, \hat{\theta}) \tag{35a}$$

$$\tilde{Y}_2 = g(Y_1, X_{2,} \tilde{\varepsilon}_2, \hat{\theta}) \tag{35b}$$

$$\vdots$$

$$\tilde{Y}_T = g(Y_{T-1}, X_{T,} \tilde{\varepsilon}_T, \hat{\theta}) \tag{35c}$$

where $\tilde{\varepsilon}_t$ is a matrix of random numbers drawn from a random numer generator or an empirical distribution function.

Define $Y = \{Y_t\}_{n\cdot TxK}$ and $\tilde{Y} = \{\tilde{Y}_t\}_{nT\times K}$ and assume that there is a criterion function defined on the difference $Y - E(\tilde{Y})$, say $L(Y - E(\tilde{Y}))$, where E is the mathematical expectation over the distribution of ε_t conditional on Y_0, X_1, ...X_T and θ. Estimates $\hat{\theta}$ are in principle obtained by maximizing *L* with respect to θ.

Assume now that bench-mark data are available in the form of population totals $\bar{y}_{kt}$ for some year t within the sampling period. Define $\bar{Y} = \{\bar{y}_{kt}\}_{1\times K}$. If the sample of individuals was drawn by simple random sampling the model estimate of this total is

$$\tilde{\tilde{Y}}_t = \frac{N}{n} J' g(Y_{t-1}, X_t, \tilde{\varepsilon}_t, \hat{\theta}) \tag{36}$$

where J is a n dimensional vector of ones. If *n* is large enough to make the effects of random variation through $\tilde{\varepsilon}_t$ small, one could maximize the estimation criterion *L* subject to the constraint $\overline{Y}_t = \tilde{\tilde{Y}}_t$.

If the external information applies to a date outside the sample period $\tilde{\tilde{Y}}_t$ has to be computed slightly differently,

$$\tilde{\tilde{Y}}_t = \frac{N}{n} J' g\left(\tilde{Y}_{t-1}, X_t \tilde{\varepsilon}_t, \hat{\theta}\right) \tag{37}$$

In the special case of a linear model this estimation problem reduces to a well-known constrained estimation problem found in textbooks. To simplify, assume there is only one endogenous variable and that the estimation criterion is the usual least-squares criterion. Then the model becomes

$$\begin{aligned} Y_t &= \left\{Y_{t-1}, X_t\right\}\theta + \varepsilon_t; \\ E\left(\varepsilon_t \middle| Y_{t-1}, X_t\right) &= 0 \end{aligned} \tag{38}$$

Let $Z_t = \left\{Y_{t-1}, X_t\right\}$, $Z = \left\{Z_t\right\}$, and $Y = \left\{Y_t\right\}$. Assume a total $\overline{y}_{t_0}$, is known for period $t_o \leq T$. The constraint then becomes

$$\frac{N}{n} J' Z_{t_0} \theta = \overline{y}_{t_0} \tag{39}$$

Minimizing the sum of squared residuals subject to this constraint gives the usual constrained least-squares estimator,

$$\hat{\theta}^* = \hat{\theta} + (Z'Z)^{-1} R' \left[R(Z'Z)^{-1} R'\right]^{-1} \left(\overline{y}_{t_0} - R\hat{\theta}\right) \tag{40}$$

where $R = \frac{N}{n} J' Z_{t_o}$ and $\hat{\theta}$ the unconstrained least-squares estimator. In this case there is thus a simple adjustment of the least-squares estimator which can be used also when the external information became available after the model was estimated. If the model is nonlinear there is in general no such simple adjustment factor. Depending on the model structure the whole model might have to be reestimated when new external information becomes available. It is straight forward to derive the variance-covariance matrix of the estimator in eq. (40), but for a nonlinear estimator it is not. If simulations are not too time consuming one might resort to sample re-use methods like boot-strapping and jack-knifing.

It could be computationally easier to align the predicitons to external information rather than to reestimate all parameters. If so one might thus prefer to do that, in particular if one is less interested in the parameter estimates as such but more in the predictions they produce. $\hat{\theta}*$ in eq. (40) is a BLUE among those estimators which satisfy the external constraint. Predictions obtained with this estimator are BLUP. Given a matrix $Z_\tau = \left\{\tilde{Y}_{\tau-1}, X_\tau\right\}$ of initial conditions the predictions become,

$$\tilde{Y}_\tau = Z_\tau \hat{\theta}* = Z_\tau \hat{\theta} + Z_\tau (Z'Z)^{-1} R' \left[R(Z'Z)^{-1} R'\right]^{-1} \left(\bar{y}_{t_0} - R\hat{\theta}\right). \tag{41}$$

The last term of this expression gives the necessary alignment of the prediction. One may note that not even in this simple linear case it is a proportional adjustment. In our notation and for the case of K=0 and only one constraint an alignment proportional to the prediction error $\left(\bar{y}_{t_0} - R\hat{\theta}\right)$ can be written

$$1 + (R\hat{\theta})^{-1} \left(\bar{y}_{t_0} - R\hat{\theta}\right); \tag{42}$$

which differs from the alignment factors obtained from eq. (41),

$$I + Z_\tau (Z'Z)^{-1} R' \left[R(Z'Z)^{-1} R' \right]^{-1} (\bar{y}_{t_0} - R\theta) \left[(Z_\tau \hat{\theta})' (Z_\tau \hat{\theta}) \right]^{-1} (Z_\tau \hat{\theta})' ; \qquad (43)$$

where $\left[(Z_\tau \hat{\theta})'(Z_\tau \hat{\theta}) \right]^{-1} (Z_\tau \hat{\theta})'$ is a generalized inverse of $(Z_\tau \hat{\theta})$. In this case each individual gets its own alignment factor. One may also note that in a model with more than one endogenous variable a constraint which applies to one variable will in general not only imply an alignment of that particular variable but also of all other variables. Furthermore, in nonlinear models there will in general not exist as simple alignment factors as in the linear case.

More or less explicitly the discussion above was based on the assumption that the sample used in the simulations had a size sufficiently large to justify the treatment of external data as exact constraints. If this is not the case one might not like the simulated total (mean) to equal the external total (mean) exactly but allow for the built in stochastic variation in the model. If the external data are estimates rather than population parameters then that is another reason not to enforce an exact equality. A natural approach to incorporate uncertain external information is that of mixed estimation, a technique which is well developed for linear models in many text books, but less developed for nonlinear models.

Model validation

If the tax and benefit legislation has been translated into computer code with sufficient detail and care and the data are detailed and accurate enough there is no need to validate a conventional static tax-benefit model without behavioral adjustments, because there is nothing to validate. However, if the simulation model includes behavioral adjustments there is a validation problem. How would one go about validating a static model? Is it at all possible? The problem with the comparative statics of a static micro-simulation model is that it does not give predictions for any specific time point or time interval, and thus, it is hard to know to what the predictions should be compared. Suppose for instance, that a labor force participation equation is estimated from a cross-section at the end of a long period of unchanged tax and benefit systems and a stable labor market. Then a major tax reform takes place. Is it a good idea to validate the predictions from this model by comparing with observed participation rates from the first, second or third, etc year after the reform?

Validation of a dynamic and dated model does not suffer from the same problem. In this case predictions have a correspondence in the real world. Model validation should proceed along two different lines. One is conventional specification testing of each single submodel in the model building phase, the other is the testing of model simulations from the entire model against external data. That is, data not used in the estimation and simulation of the model. In validating the model one would like to take account of the fact that the simulations are subject to stochastic errors. These errors originate from two sources. One is the stochastic model structure. Events are generated by invoking random number generators. The other source is the set of parameter estimates. We do not know the true parameters only error prone estimates.

For a model not to big and complex in structure it might be feasible to derive an analytical expression for the variance-covariance matrix of the simulations which takes both sources into account, for an example see Pudney & Sutherland (1996). In general micro simulation models are so complex that analytical solutions are unlikely. Given the parameter estimates the uncertainty generated by the model as such can be evaluated if a simulation is replicated with new random number generator seeds for each replication. There is a trade off between the number of replications needed and the sample size. The bigger sample the fewer replications.

To evaluate the uncertainty which arises through the parameter estimates the distribution of the estimates can be approximated by a multivariate normal distribution with mean vector and covariance matrix equal to that of the estimated parameters. By repeated draws from this normal distribution and new model simulations for each draw of parameter values an estimate of the variability in the simulations due to the uncertainty about the true parameter values can be obtained.

To avoid the normal approximation one could consider using sample re-use methods. For instance, by jack-knifing or boot-strapping a set of replicated estimates of the model parameters can be obtained. Each replication can be used in one or more simulation runs, and the variance of these simulations will capture both the variability in parameter estimates and that due to the random nature of the model.

Even if model simulations do not deviate more from new bench marks than is normal given the stochastic properties of the model, one might like to improve the precision of the parameter estimates by updating or calibrating them to this new bench mark information. How this can be done was discussed above. If the simulations deviate significantly from the new bench marks, that is an indication of a misspecified model. In this situation it would seem improper just to calibrate the parameters to the new bench marks or align the model simulations to them. A re-specification of the model might be necessary.

Finally we should also note that validation need not only be done against bench marks like means and totals. If frequency distributions or measures of dispersion and correlation are available they could also be used. As noted above a micro simulation model is likely to have a number of simplifying assumptions about independence of variables, which might cause variances and in particular correlations to decrease over time. Validation against observed correlations and dispersions will then prove useful.

Much of the total error in simulated values will come from the choice of a particular model structure or specification. Sensitivity analysis is an approach to assess the importance of this source of error. As pointed out in Citro & Hanushek(1997) p. 155 "sensitivity analysis is a diagnostic tool for ascertaining which parts of an overall model could have the largest impact on results and therefore are the most important to scrutinize for potential errors that could be reduced or eliminated". If simple measures of the impact on key variables from marginal changes in parameters and exogenous entities could be computed they would potentially become very useful.

5. Conclusions

After the dismal experiences with structural macro models we had the hope that modeling at the micro level and using large samples of micro data would yield estimated relations with some stability and scope. This hope has only been met to a limited extent (see for instance the discussion of models to capture work incentives in Atkinson & Mogensen, 1993). It is hard to know if this is the result of the nonexistence of stable micro relations, that the behavior of economic agents changes as the result of new policies, new institutions and other external changes, or of insufficient data and inadequate research approaches in economics (for a discussion see Klevmarken, 1994), or that the research process simply has to take more time. Behavioral modeling in the micro simulation context cannot be expected to go much beyond the state of art in economics. In each module of a large micro-simulation model modeling meets with the same difficulties as in more conventional economic modeling, but in addition it has the difficulty of making the different modules fit together. There are obvious problems when modules have to be tested and estimated on different data sets, but there is also a requirement of an internal consistency of the model structure. For instance, if one module needs a particular explanatory variable, then another module is needed to simulate it such that the simulated values can be fed into the first module. To handle these problems the model builder needs a strategy as to the general model structure, as discussed above. A piecemeal approach in which one starts with one module and then takes decisions about subsequent modules depending on the outcome of the research for the first is likely to lead to inconsistencies and to force the model builder to painful compromises for practical purposes.

As pointed out in Citro & Hanushek(1997) p. 142 “Such a modeling framework is also useful because it can help structure related analytical work. The effort to develop and apply a large-scale micro-simulation model will invariably identify behavioral interactions and processes that need to be better understood. It will also help determine which parameters are crucial for analysis and which are less important, and it can suggest how concepts and variables should be consistently defined and measured to be useful for modeling purposes.”

It is also obvious that the availability of a rich data source of micro data will reduce the need to use supplementary data sets and thus greatly facilitate modeling. Depending on the purpose it would seem essential to have at least the key policy and effect variables included in the same data set.

Assumptions of independence or conditional independence should be limited to relations, which are of second order importance to the uses of the micro-simulation model. If the dynamics of behavioral adjustments is important, which is almost always the case in policy simulations and evaluations, then panel data are needed. The large household panel data sets collected in several countries are thus essential for the construction of general micro-simulation models of the household sector.

Modeling for regions larger than a country, EU for instance, would in principle require comparable data collected in all countries. Separate but comparable surveys in each country could be used to design comparable models for each country, which could be run one by one. Such an approach would make feasible an analysis of the same policy carried out in each country separately, but it would not permit the analysis of any interacting effects across boarders. If, for instance tax policies and social policies in one country are likely to attract or detract workers from another country, then a data collection design is needed which permits the survey people to follow respondents from one country to another to make feasible an analysis of the region wide policy effects mediated by migration or other across boarder activities.

Given that the above mentioned difficulties can be handled in a satisfactory way micro-simulation offers in principle opportunities to submit behavioral models to stronger tests than the usual diagnostic and specification testing done for each module separately. In addition to these tests a micro-simulation model can be tested by comparing the simulated results with external data. Because simulated data can be aggregated, the data used to ”calibrate” against could either be micro data or aggregate data, for instance from the national accounts. It is a practical problem that these data need apply to the same population and observational units as the micro-simulation model and they also need to comply with the same variable definitions.

The methodology for this ”calibration” is not fully developed. In particular there are a few issues which should be studied. First, the choice of criterion for a good model, second the inference theory needed to decide if the simulated (predicted) data lie within reasonable confidence bands from the observed data, and third, methods to evaluate the marginal influence of each parameter on the simulation results. It would be very useful to know which parameters have the most influence on the simulated results. A fourth issue is the estimation theory needed to incorporate new benchmark data.

Most of the modeling done in micro-simulation is of the type ”average behavior with random deviations”. Conventional econometric models have been plugged into micro-simulation models. As indicated above micro-simulation offers opportunities to deviate from the paradigm of average behavior and allows for systematic differences in behavior, for instance, individual preference parameters estimated from panel data. One should probably also explore more the techniques to copy ”donors” by the random assignment approach, which avoids unnecessary restrictive assumptions about functional forms.

Finally a word about the role of micro-simulation as part of a research strategy. In their resent evaluation of micro-simulation and alternative approaches to assess policies for retirement income the U.S. Panel on Retirement Income Modeling (Hanushek & Maritato, 1996 and Citro & Hanushek, 1997) recommended that the relevant agencies should consider the development of an individual-level micro-simulation model as an important long-term goal, but that the construction of such a model would be premature until better data, research knowledge, and computational methods are available. This might be a sensible recommendation in this particular case considering the long duration of the economic process that need modeling and the data situation in the United States. A different conclusion could have been reach for another country. However, also in the case of the United States this recommendation misses the importance of allowing a micro-simulation project organize both modeling efforts and data collection.

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Table 1. **Static models with behavioral modeling**

Model	**References**	**Country**	**Behavior**
STATS	Bridges & Johnston (1976)	USA	Program participation
MATH	Beebout (1977) Kidd (1979) Doyel & Whitemore (1982) Beebout (1986)	USA	Demographics and labor supply, program participation
KGB	Betson et.al. (1982) O'Reilly (1977)	USA	Labor supply, program participation
TAXSIM	Feldstein (1983)	USA	Labor supply, demand for housing, charitable giving
MICSIM	Merz (1989a,b, 1990b) Merz & Buxmann (1990)	FRG	Labor supply
	Bekkering, Grift & Siegers (1986)	NE	Labor supply
	Chernic et.al.(1987)	USA	Health insurance and demand for medical services
	Van Soest (1988)	NE	Labor supply
	Kapteyn, Woitties & ten Hacken (1989)	NE	Labor supply
	Grift et.al. (1991)	NE	Labor supply
TAXBEN/ SPAIN	Giles (1995) Johnson, Stark & Webb (1990) Duncan (1991)	UK	Labor supply
CNAF	Grignon & Pennec (19)	F	Fertility, housing
INDICE	Patrizii & Rossi (1991)	I	Household expenditures - indirect taxes
INDIMOD	Baldini (1995)	I	Household expenditures - indirect taxes
ASTER	Decoster, Rober &Van Dongen (1994)	B	Household expenditures- indirect taxes
POLIMOD	Redmond, Sutherland & Wilson (1995)	UK	Labor supply

Table 2. General dynamic models with behavioral relations

Model	References	Country
DYNASIM	Orcutt et.al. (1976a,b)	USA
DYNASIM II	Johnson & Zedlewski(1982) Johnson et.al.(1983) Zedlewski(1990) Wertheimer II et.al (1986)	USA
MICROSIM	McKay (1978)	USA
MICROSIM/MASS	Orcutt & Smith (1979)	USA
Sfb3-MSM	Helberger (1982) Hain & Helberger (1986) Galler & Wagner (1986) Galler (1989, 1994)	FRG
CORSIM	Caldwell (1988,1993)	USA
HARDING	Harding (1990, 1993)	UK, AUS
DEMOGEN	Wolfson (1990)	CAN
LIFEMOD	Falkingham & Lessof(1991, 1992)	UK
MOSART	Andreassen et.al.(1992, 1993, 1994) Andreassen (1993)	N
MICROHUS	Klevmarken et.al. (1992) Klevmarken & Olovsson (1996)	S
DYNAMOD	Antcliff (1993)	AUS
NEDYMAS	Nelissen (1994)	NE

Table 3. Specialized models with behavioral relations

Model	References	Country	Behavior
RFV-ATP	Eriksen (1973)	S	Life cycle earnings demographic transitions
	Vroman (1980)	USA	Work and unemployment
Mikropolis	Beckering, Schaaijk, Verkode &Waijers (1989)	NE	Labor supply labor demand
PRISIM	Kennell & Sheils (1986, 1990)	USA	Decision to retire and accept pension benefits
	Atherton et.al. (1990)	USA	Local residential telephone demand
SPEND	Baker (1991)	UK	Energy demand
SPIT	Baker & Symons (1991)	UK	Household consumption - indirect taxation
	Erksoy (1992a,b, 1994)	CAN	Unemployment
	Baekgaard (1993)	DK	Demand for child care
	Merz (1993)	FRG	Market and nonmarket labor supply
	Bekkering (1995)	NE	Labor market (demographic and educational transitions by constant probabilities)
	Symons & Warren (1996)	AUS	Household consumption behavior
TOPSIM I	Holm et.al. (1996)	S	Regional demography
FAMSIM	Lutz(1997)	A	Household formation and dissolution
	Fransson (1997)	S	Household formation and housing market